

Anderson localization in Hubbard ladders

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The effect of a weak random potential on two-leg Hubbard ladders is investigated. The random potential is shown to induce Anderson localization except for attractive enough interactions, suppressing completely d-wave superconductivity. These localization effects remain very strong even for many ladders coupled by Josephson coupling. Both dc and ac conductivities and localization lengths are obtained. Consequences for the superconducting ladder compound $\text{Sr}_x\text{Ca}_{14-x}\text{Cu}_{24}\text{O}_{41+\delta}$ are discussed.

The two-chain Hubbard ladder is a toy model of a metal with a spin-gap, reminiscent of the metallic phase of underdoped cuprates. Recent experiments[1] on the superconducting transition in $\text{Sr}_{14-x}\text{Ca}_x\text{Cu}_{24}\text{O}_{41}$ have been interpreted in the framework of this model. However, impurities are only considered as a source of holes and not a source of random potential although it is known that an infinitesimal disorder localizes all electronic states in one dimension. It is therefore important to investigate the disordered Hubbard ladder. The disordered Hubbard ladder is defined by the Hamiltonian:

$$\begin{aligned} H = & -t_{\parallel} \sum_{i,p,\sigma} (c_{i+1,p,\sigma}^{\dagger} c_{i,p,\sigma} + c_{i,p,\sigma}^{\dagger} c_{i+1,p,\sigma}) \\ & - t_{\perp} \sum_{i,\sigma} (c_{i,1,\sigma}^{\dagger} c_{i,2,\sigma} + c_{i,2,\sigma}^{\dagger} c_{i,1,\sigma}) \\ & + U \sum_{i,p} n_{i,p,\uparrow} n_{i,p,\downarrow} + \sum_{i,p} \epsilon_{i,p} n_{i,p} \end{aligned} \quad (1)$$

Where ϵ is a random potential with $\overline{\epsilon_{i,p}\epsilon_{j,q}} = D\delta_{i,j}\delta_{p,q}$. For zero disorder ($D = 0$), bosonization techniques allow to reduce the Hamiltonian (1) to a Hamiltonian with two bosonic charge modes and two bosonic spin modes. Among those, only the mode corresponding to total charge excitations is gapless. The ladder is thus in a metallic state with a spin gap Δ . The total charge mode is completely characterized by the velocity of charge excitations $u_{\rho+}$ and an exponent $K_{\rho+}$. for attractive interactions, $K_{\rho+} > 1$

and for repulsive interactions $K_{\rho+} < 1$. With attractive interactions, s-wave superconducting fluctuations are dominant, whereas for repulsive interactions, d-wave superconducting fluctuations are dominant [2].

The effect of a weak disorder $D \ll \Delta$ can be considered in a Renormalization Group analysis [3]. For $K_{\rho+} < 3/2$, the disorder is relevant and the system is in an Anderson localized phase. In particular, the d-wave superconducting “phase” of the ladder is unstable with respect to infinitesimal disorder, and the s-wave superconducting “phase” is stable only for attractive enough interactions.

The localization length $L_{loc.}$ and both the dc and ac transport can also be computed using the same RG procedure [3] and are shown respectively on Fig. 1 and Fig. 2. The temperature dependent conductivity is

$$\sigma_{dc}(T) \sim T^{2-2K_{\rho+}} \quad (2)$$

in the regime $k_B T \gg \frac{\hbar u_{\rho+}}{L_{loc.}}$. In the localized regime and for $k_B T \ll \frac{\hbar u_{\rho+}}{L_{loc.}}$, the conductivity behaves as $\sigma_{dc}(T) \sim \exp(-(T_0/T)^{1/2})$.

As can be seen from Fig.1 and (2) there is a maximum in σ_{dc} as a function of T for attractive interactions which occurs when the thermal coherence length is of the order of magnitude of the localization length $T \sim \frac{u_{\rho+}}{k_B L_{loc.}}$. This maximum is therefore a remnant of the s-wave superconductivity in the pure system. Conversely, there is *no remnant* of the d-wave superconductivity of the pure system for repulsive interactions. Although

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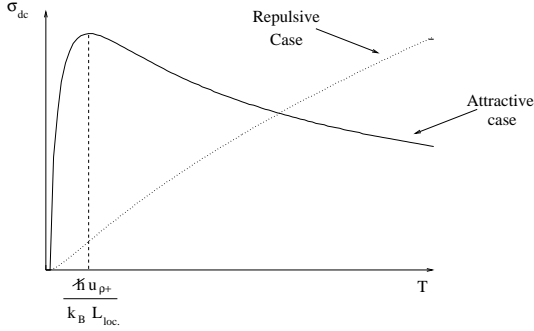


Figure 1. The behavior of d.c. conductivity as a function of temperature for repulsive and for attractive interactions.

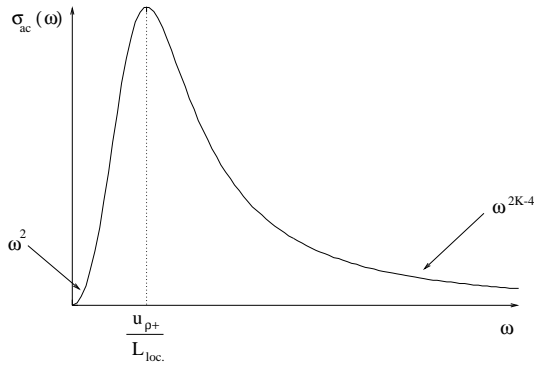


Figure 2. The behavior of a.c. conductivity (for $T = 0$) as a function of frequency in the localized regime $K = K_{\rho+} < 3/2$.

the pure system has dominant d-wave superconductive fluctuations the transport properties in presence of disorder resemble the one of an insulator. These predictions on transport could be tested in ladder compounds but also in quantum wire systems.

In the single ladder, d-wave superconductivity resulting from purely repulsive interactions is unstable in the presence of infinitesimal disorder. To determine if this result persists in a system of coupled ladders we have performed a mean field treatment of Josephson coupled ladders [3]. Such a system would have a genuine ordered d-wave phase and a finite critical temperature in the pure case. Even in the presence of interladder Josephson coupling d-wave superconductivity is very unstable with respect to non magnetic disorder, leading again to a destruction of d-wave superconductivity for realistic disorder strength. Let us note that this instability is much stronger than a simple pair-breaking effect, and is due here to the localization effect coming from the *one-dimensional* nature of the ladder [3].

Both the temperature dependence of the conductivity of a purely one-dimensional ladder model and the extreme sensitivity of T_c to impurities even for coupled ladders, strongly suggests [3] that the superconducting transition [1] in $\text{Sr}_x\text{Ca}_{14-x}\text{Cu}_{24}\text{O}_{41+\delta}$ cannot be explained in terms of stabilization of one-dimensional d-wave fluctuations by interchain coupling and that a better theory of superconductivity in these compounds should start from a truly two-dimensional limit.

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